

**Numerical Modelling of Electromagnetic Wave
Propagation and Scattering: High-Order Schemes,
Impedance Boundary Conditions and Cole-Cole
Dielectrics**

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14. ABSTRACT Paper [1] was listed in the report for Grant F49620-99-1-0072 as "to appear" and the work presented there was described previously. In paper [2] we presented an analysis of the perfectly matched layer in cylindrical coordinates discretized with a staggered second-order accurate finite difference time domain method. In paper [3] we conclusively addressed the long standing issue of the long-time stability of the unsplit Perfectly Matched Layer. In paper [4] we examine the short- and long-time response of a Cole-Cole dielectric halfspace subjected to a delta-function incident pulse. We find that the Cole-Cole impulse response is infinitely smooth at the wavefront (short-time) in contrast to the case of the Debye impulse response that is discontinuous at the wavefront.				
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1 Technical Summary of Research Accomplished

We will first briefly describe the research results that have already been published in the scientific literature, and then we will provide some details of the remaining publications that resulted from the research effort and are still in preparation or have been submitted.

1.1 Summary of Published Research Accomplishments

Paper [1] was listed in the report for Grant F49620-99-1-0072 as "to appear" and the work presented there was described previously.

In paper [2] we presented an analysis of the perfectly matched layer in cylindrical coordinates discretized with a staggered second-order accurate finite difference time domain method. For fixed discretization parameters, layer width, and a quadratic loss function, we found that the numerical reflection produced by the discrete layer is accurately predicted by the infinite resolution reflection coefficient for $\sigma_{max} \in [0, \sigma_{max}^c]$, where σ_{max} is the maximum value of the absorption parameter in the layer. We also find that the finite resolution reflection coefficient achieves its minimum value at a $\sigma_{max}^m > \sigma_{max}^c$. Numerical experiments validated the analysis. Figure 1 below shows that the reflection produced by the discrete cylindrical PML converges to zero with increasing σ_{max} , again in accordance with the exact reflection coefficient (labelled as Equation (15) on the graph). We also see that, for a fixed spatial mesh size, $\Delta\rho$, the reflection converges to zero with increasing layer depth d_ρ , again in accordance with (15). The most important observation is the fact that (15) is a good predictor of the numerical reflection as a function of σ_{max} when that parameter is in a range $(0, \sigma_{max}^c]$. We expect to have obtained similar behavior had we used d_ρ as the variable parameter. A $\times 2$ mesh refinement (with all other parameters fixed) indicates an enlargement of the range of σ_{max} that allows (15) to be an accurate predictor of performance. Significantly, Figure 1 shows that Equation 22, the main result of [2], is an extremely accurate predictor of discrete PML reflection.

In paper [3] we conclusively addressed the long standing issue of the long-time stability of the unsplit Perfectly Matched Layer. Research groups at Brown University and elsewhere had in the past noticed linear instabilities in long-time simulations with the unsplit PML and had proposed remedies. Such remedies, however, always resulted in the layer not being perfectly matched. In [3] we showed how to eliminate the undesirable long-time linear growth of the electromagnetic field in a class of unsplit Perfectly Matched Layers (PML) typically used as Absorbing Boundary Conditions in Computational Electromagnetics codes. For

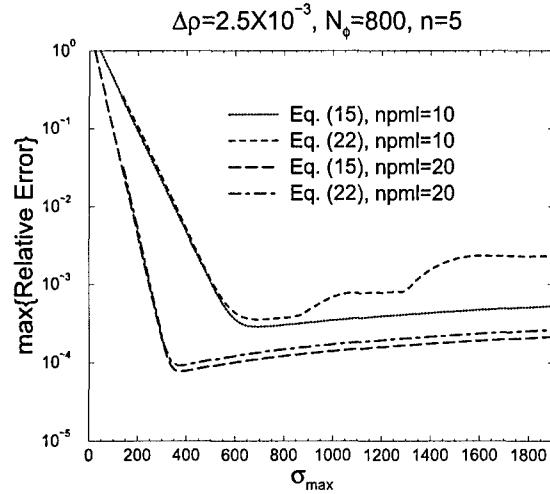
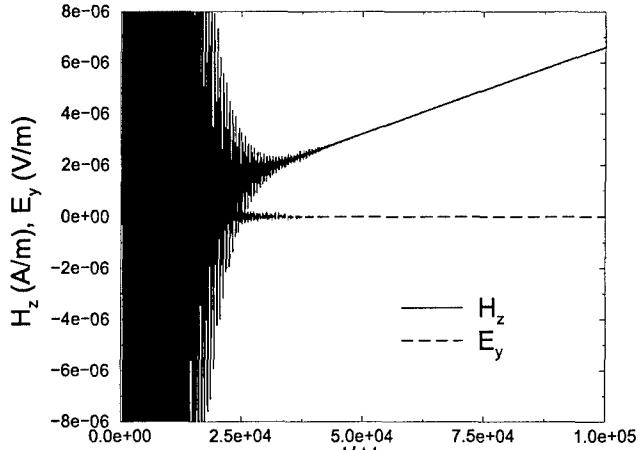


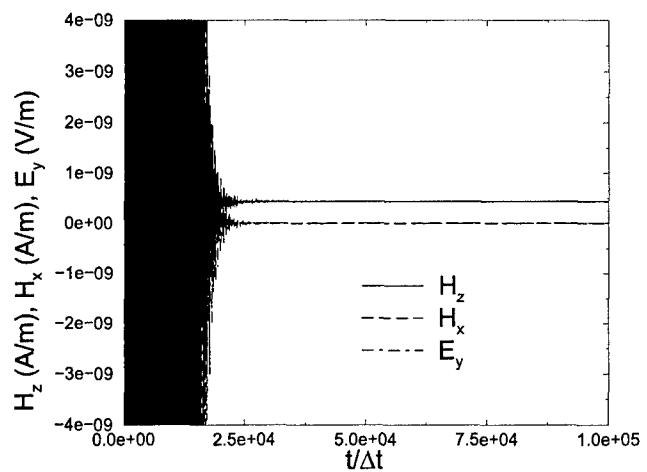
Figure 1: The maximum of the actual numerical error over the time interval $[0, 15]$ as a function of σ_{max} ; The dashed line corresponds to a $\times 2$ mesh refinement of the $npml = 10$ case. The straight line segments represent the corresponding $|R_n^{\Omega_{pml}}|$.

the resulting new PML equations we gave energy arguments that showed the fields in the new layer are bounded by a time-independent constant hence the layer is long-time stable. Numerical experiments confirmed the elimination of the linear growth, and the long-time boundedness of the fields. For example, Figure 2-(a) illustrates the late-time response of the axial Magnetic field, H_z , in the usual unsplit PML; it is observed that it exhibits a linear growth in time while the remaining field components tend to zero in full accordance with our analysis in [3]. This simulation was repeated with $\gamma = 0.08$. The late-time fields are shown in Figure 2-(b). Now, all field components remain bounded by a time-independent constant in full agreement with our energy proofs in [3]. Further, Figure 2-(b) shows that when $\gamma = 0.08$ only the axial magnetic field tends to a constant while the other field components decay to zero. Again, this is in agreement with our energy considerations given in Section 4 of [3]. Significantly, and in contrast to previous work on this topic by others, our modification of the standard Unsplit PML eliminates the long-time instability *while maintaining* the perfectly matched property of the resulting layer.

A small amount of time was invested by the PI during the academic years covered by this report on a collaboration with colleagues which aims to control fluid interfaces with electric fields. Papers [7]-[8] investigate the stability of a thin two-dimensional liquid film when a uniform electric field is applied in a direction parallel to the initially flat bounding



(a)



(b)

Figure 2: a) The late-time linear growth of the axial PML field, $H_z(31, 59, t)$. The $H_x(31, 59, t)$ field behaves exactly like the graphed electric field $E_y(31, 59, t)$. This case, $\gamma = 0$, represents the standard unsplit PML. b) $\gamma = 0.08$; the late-time linear growth of H_z has been removed.

fluid interfaces. We considered the distinct physical effects of surface tension and electrically induced forces for inviscid and viscous, incompressible, nonconducting and conducting, fluids.

1.2 Summary of Research Results to be Published

In paper [4] we examine the short- and long-time response of a Cole-Cole dielectric half-space subjected to a delta-function incident pulse. Our purpose is to contrast the short- and long-time impulse response of the Cole-Cole dielectric model to that obtained previously by the PI for the Debye model. The purpose for this is to determine whether the time-domain waveforms obtained in a Time-Domain Reflectometry (TDR) experiment could serve as a means to determine the most appropriate frequency-domain model for the data at hand. Time-domain waveforms in TDR experiments are routinely measured and analyzed. The modeled short- and long-time response of the experimentally examined dielectric subjected to any physically realizable pulse (hence measurable in a TDR experiment) will be obtained by convolution with the appropriate impulse response function. We find that the Cole-Cole impulse response is infinitely smooth at the wavefront (short-time) in contrast to the case of the Debye impulse response that is discontinuous at the wavefront. Also, we find that the location of the peak of the main (late-time) response in the Cole-Cole dielectric occurs at an earlier space-time location than that found for the Debye dielectric main response.

In [4] we formulate the time-domain problem for the impulse response of the Cole-Cole medium and reduce to a signaling problem in a dielectric half space ($x \geq 0$) for the electric field which is given by the following Bromwich integral

$$E(x, t) = \frac{1}{2\pi i} \int_{\zeta-i\infty}^{\zeta+i\infty} e^{s[t - \frac{x}{c_\infty} \sqrt{\frac{s^\alpha + \beta}{s^\alpha + \beta\gamma}}]} ds, \quad t \geq \frac{x}{c_\infty}, \quad (1)$$

where $-\pi < \arg(s) \leq \pi$, $\operatorname{Re} \sqrt{\frac{s^\alpha + \beta}{s^\alpha + \beta\gamma}} > 0$, $\beta = \frac{\epsilon_s T_p^\alpha}{\epsilon_\infty \tau^\alpha}$, $\gamma = \frac{\epsilon_\infty}{\epsilon_s}$, and T_p is a time scale characterizing the any other signaling data which will be convolved with $E(x, t)$ above to obtain the measured response. Closing the Bromwich contour with a semi-circle to the right of $s = \zeta$ we obtain $E(x, t) = 0$, $t < \frac{x}{c_\infty}$, as the integrand exhibits no singularities in that region. For the branch chosen, the square root in (1) does not introduce additional singularities (recall $0 < \alpha < 1$) since the argument of the roots of the numerator and denominator is $\arg(s) = (\pi + 2k\pi)/\alpha$; $k = 0, \pm 1, \dots$, i.e., these roots are outside the chosen principal branch. In the wavefront region ($t \rightarrow \frac{x^+}{c_\infty}$) a large- s expansion of the bracketed

expression in (1)

$$(t - \frac{x}{c_\infty}) + \frac{x}{c_\infty} (1 - \sqrt{\frac{s^\alpha + \beta}{s^\alpha + \beta\gamma}}) \approx (t - \frac{x}{c_\infty}) - \frac{x}{c_\infty} \frac{\beta(1 - \gamma)}{2} s^{-\alpha} \quad (2)$$

results in

$$E(x, t) \approx \frac{1}{2\pi i} \int_{\zeta-i\infty}^{\zeta+i\infty} e^{-As^{1-\alpha}} e^{s(t - \frac{x}{c_\infty})} ds, \quad (3)$$

where $A = \frac{x}{c_\infty} \frac{\beta(1-\gamma)}{2} > 0$. Due to $0 < 1 - \alpha < 1$, we determine that

$$\lim_{t \rightarrow \frac{x}{c_\infty}^+} \frac{d^n}{dt^n} E(x, t) = 0, \quad n \geq 0, \quad \text{since} \quad \lim_{s \rightarrow \infty} s^n e^{-As^{1-\alpha}} = 0, \quad n \geq 0. \quad (4)$$

Thus, in contrast to the Debye medium model wavefront response, the response is infinitely smooth at the wavefront. For the special case $\alpha = 1/2$, standard Laplace transform tables allow us to invert (3) via

$$\mathcal{L}^{-1}\{e^{-\frac{x}{c_\infty} \frac{\beta(1-\gamma)}{2} s^{1/2}}\} = \frac{1}{4\sqrt{\pi}} \frac{\beta(1-\gamma)}{c_\infty} \frac{x}{t^{3/2}} e^{-\frac{\beta^2(1-\gamma)^2 x^2}{16c_\infty^2 t}}, \quad t > 0, \quad (5)$$

and obtain the short-time response for an arbitrary pulse f incident on the half-space at $x = 0$

$$E(x, t) \approx \frac{1}{4\sqrt{\pi}} \frac{\beta(1-\gamma)}{c_\infty} x \int_{x/c_\infty}^t f(t-\xi) \frac{1}{(\xi - x/c_\infty)^{3/2}} e^{-\frac{\beta^2(1-\gamma)^2 x^2}{16c_\infty^2 (\xi - x/c_\infty)}} d\xi. \quad (6)$$

In fact, by collapsing the Bromwich contour in (3) onto the branch cut defined earlier we can write $\Psi_\alpha(t) = \mathcal{L}^{-1}\{e^{-s^{1-\alpha}}\}$, $t > 0$, where

$$\Psi_\alpha(t) = \frac{1}{\pi} \int_0^\infty e^{-tr} e^{-r^{1-\alpha} \cos[\pi(1-\alpha)]} \sin(r^{1-\alpha} \sin[\pi(1-\alpha)]) dr. \quad (7)$$

Then, (7) and the translation theorem allows us to write an integral expression for $E(x, t)$ for any $0 < \alpha < 1$ as a convolution of $f(t)$ with

$$\mathcal{L}^{-1}\{e^{-As^{1-\alpha}}\} = A^{-\frac{1}{1-\alpha}} \Psi_\alpha(A^{-\frac{1}{1-\alpha}}(t - \frac{x}{c_\infty})), \quad t > \frac{x}{c_\infty}. \quad (8)$$

For demonstration purposes we employed Mathematica to evaluate the integral in (7) for $0 < t - \frac{x}{c_\infty} < 2$; we present in Figures 3 and 4 the wavefront response for the case $A = 1$ and various values of α ; as $\alpha \rightarrow 1^-$ the right hand side of (7) approaches a delta function in time

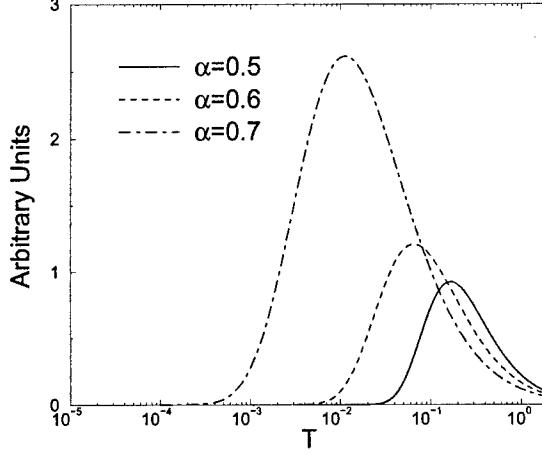


Figure 3: Wavefront response for $\delta(t)$ signaling data; $T = t - \frac{x}{c_\infty}$.

and the numerical evaluation of the integral fails. We verified the results shown in Figures 3-4 by using Mathematica to evaluate the following alternative representation of $\Psi_\alpha(t)$,

$$\Psi_\alpha(t) = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{t^{-(1-\alpha)k-1}}{\Gamma(1+k)\Gamma(k(\alpha-1))}. \quad (9)$$

Finally, we note that in the case of unit step-function signaling data ($F(s) = 1/s$) the electric field response exhibits self-similarity, i.e., $E(x, t) = \Phi_\alpha(\xi)$, where $\xi = A^{-\frac{1}{1-\alpha}}(t - \frac{x}{c_\infty})$ is the similarity variable and $\Phi_\alpha(\xi) = \int_0^\xi A^{-\frac{1}{1-\alpha}} \Psi_\alpha(A^{-\frac{1}{1-\alpha}}t) dt$ (using $\Psi_\alpha(0) = 0$ and the integration property of the Laplace transform).

In the $t > \frac{x}{c_\infty}$ region we evaluate (1) using the saddle point (steepest descent) method. We first rewrite (1) as

$$E(x, t) = \frac{1}{2\pi i} \int_{\zeta-i\infty}^{\zeta+i\infty} e^{\lambda Q(s, \theta)} ds, \quad (10)$$

where $\theta = \frac{c_\infty t}{x}$, $Q(s, \theta) = s[\theta - \sqrt{\frac{s^\alpha + \beta}{s^\alpha + \beta\gamma}}]$, and $\lambda = x/c_\infty$ is a large parameter. We obtain the location of the saddle points, $s = \bar{s}$, by setting $\frac{\partial Q(s, \theta)}{\partial s} = 0$ and solving for s . The following equation holds for the saddle points:

$$\theta = \sqrt{\frac{s^\alpha + \beta}{s^\alpha + \beta\gamma}} [1 - \frac{\alpha\beta/2}{s^\alpha + \beta} + \frac{\alpha\beta\gamma/2}{s^\alpha + \beta\gamma}]. \quad (11)$$

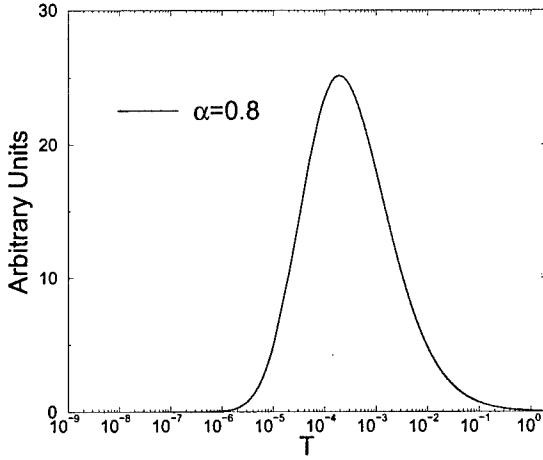


Figure 4: Same as Figure 3.

By rationalizing (11) we obtain a fourth order polynomial in s^α which we solve for a representative set of parameters using Matlab. The roots are shown in Figure 5. As θ increases from 1, the positive root s^α decreases from ∞ . When $\theta = 1/\sqrt{\gamma} = c_\infty/c_0$, $s^\alpha = 0$. For $\theta > c_\infty/c_0$ all real roots of (11) are negative. The complex roots of (11) always lie in the second and third quadrants. Consequently, on the principal sheet defined by the branch cut introduced earlier by s^α , only the positive real roots of (11) survive as saddle points. Therefore, for each value of $1 \leq \theta < c_\infty/c_0$ there corresponds one saddle point $\infty > \bar{s} > 0$ on the real axis. For $\theta \geq c_\infty/c_0$ the saddle point first coalesces with the branch point at the origin and then moves into the branch cut, hence the Bromwich contour is no longer equivalent to a steepest descent contour. In the region $1 < \theta < c_\infty/c_0$ we apply the saddle point method using $\lambda = x/c_\infty$ as our large parameter. A small- s expansion of $Q(s, \theta)$,

$$Q(s, \theta) \approx s\left(\theta - \frac{1}{\sqrt{\gamma}} + \frac{1-\gamma}{2\beta\gamma^{3/2}}s^\alpha\right), \quad (12)$$

allows us to obtain an approximation to the saddle point, i.e.,

$$\bar{s} = B^{\frac{1}{\alpha}}\left(\frac{1}{\sqrt{\gamma}} - \theta\right)^{\frac{1}{\alpha}}, \quad (13)$$

where $B = \frac{2\beta\gamma^{3/2}}{(1-\gamma)(1+\alpha)}$. We find that $\frac{\partial^2 Q(\bar{s}, \theta)}{\partial s^2} = \alpha B^{-\frac{1}{\alpha}}\left(\frac{1}{\sqrt{\gamma}} - \theta\right)^{1-\frac{1}{\alpha}} > 0$ hence the local steepest

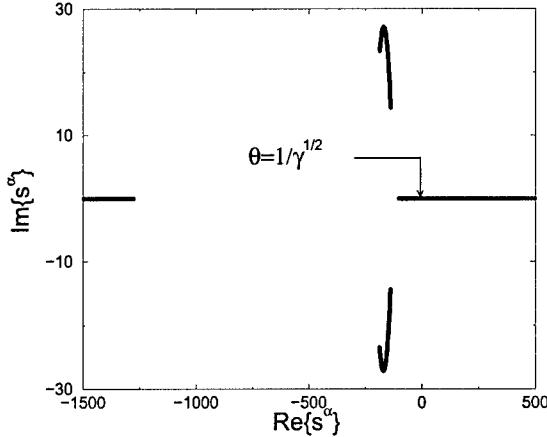


Figure 5: Motion of the roots of (11) for $1 < \theta \leq \frac{5}{\sqrt{\gamma}}$ and $\alpha = 0.7$.

descent directions at \bar{s} are $\arg(s - \bar{s}) = \frac{\pi}{2}, \frac{3\pi}{2}$. We obtain the result

$$E(x, t) \approx \frac{e^{\lambda Q(\bar{s}, \theta)}}{\sqrt{2\pi\lambda Q_{ss}(\bar{s}, \theta)}} \times \left[1 + \frac{(1-\alpha)(1+2\alpha)}{24\alpha\lambda B^{1/\alpha}(\frac{1}{\sqrt{\gamma}} - \theta)^{1+\frac{1}{\alpha}}} \right]. \quad (14)$$

In Figure 6 we plot the leading order term of (14) as a function of α . We determine that the peak of the long-time response does not occur on the sub-characteristic ray, $x = c_0 t$, as in the case of the Debye medium. Also, we notice that the leading-order result breaks down, i.e., $E(x, t) = 0$, for $\theta = c_\infty/c_0$ and $\alpha < 1$. This can be explained by noticing that the second term in (14), the correction, diverges at $\theta = c_\infty/c_0$. This is expected as the derivation of (14) does not take into account the coalescence of the saddle point with the branch point at $s = 0$. However the result is useful; Figures 7-8 show a comparison between the leading order saddle point method result and an evaluation of (10) for $x/c_\infty = 100$ using Mathematica.

In paper [5] we incorporate the Cole-Cole model in the FD-TD scheme. As a first step in that direction we developed a numerical method to solve the following fractional differential equation initial value problem for the polarization \mathbf{P}

$$\tau^\alpha \frac{d^\alpha \mathbf{P}}{dt^\alpha} + \mathbf{P} = (\epsilon_s - \epsilon_\infty) \mathbf{E}; \quad t > 0, \quad \mathbf{P}(0) = 0$$

where

$$\frac{d^\alpha \mathbf{P}}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\mathbf{P}'(\ell)}{(t-\ell)^\alpha} d\ell$$

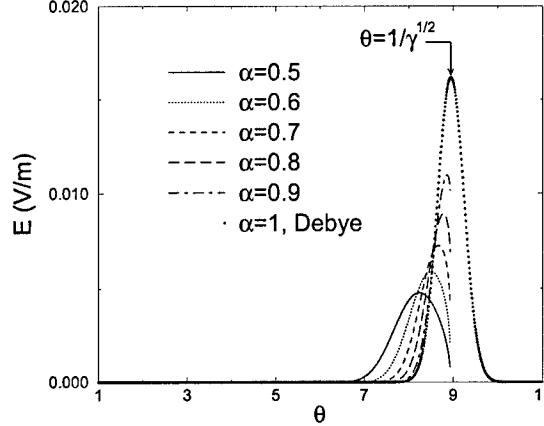


Figure 6: Leading order asymptotic response at $x/c_\infty = 100$ as a function of α .

is the Caputo fractional derivative of order $\alpha \in (0, 1)$. We use

$$\Gamma(\alpha) = \int_0^\infty e^{-z} z^{\alpha-1} dz \text{ and } \Gamma(\alpha)\Gamma(1-\alpha) = \frac{\pi}{\sin \pi\alpha}$$

and a simple change of variables, $z = \xi^2(t - \ell)$, in the Caputo derivative to show:

$$\frac{d^\alpha \mathbf{P}}{dt^\alpha} = \frac{2 \sin \pi\alpha}{\pi} \int_0^\infty \psi(\xi, t) d\xi, \quad t > 0$$

where $\psi(\xi, t)$ satisfies a non-homogeneous ODE IVP

$$\frac{d\psi}{dt} + \xi^2 \psi = \xi^{2\alpha-1} \frac{d\mathbf{P}}{dt}; \quad t > 0, \quad \psi(\xi, 0) = 0; \quad 0 \leq \xi < \infty.$$

For accurate results we decompose $\int_0^\infty d\xi = \int_0^1 d\xi + \int_1^\infty$ and use the small- ξ behavior $\psi(\xi, t) \sim y^{2\alpha-1} \mathbf{P}(t)$ and the large- ξ behavior $\psi(\xi, t) \sim y^{2\alpha-3} \mathbf{P}'(t)$, determined from the above ODE, along with Gaussian and Laguerre integration. The discretization in ξ is then followed by a discretization in t using the trapezoidal rule; the result is an $O(\Delta t^2)$ accurate system of $N_g + N_l + 1$ equations that must be solved along with the FD-TD scheme. Typically, $N_g = N_l = 10 - 15$. We show tests of our algorithm for the problem

$$\frac{d^\alpha \mathbf{P}}{dt^\alpha} + \mathbf{P} = 0, \quad P(0^+) = 1$$

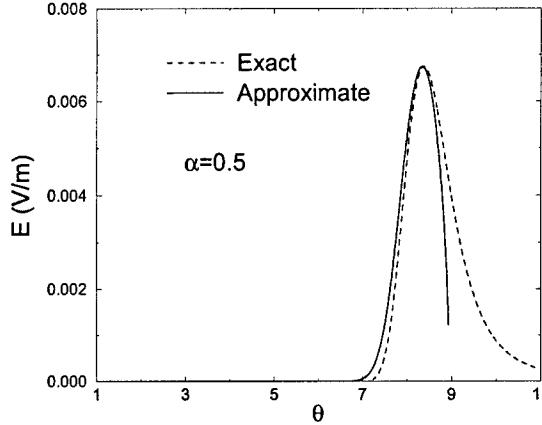


Figure 7: Comparison of Equation (14) and (10) for $x/c_\infty = 100$.

The exact solution of the above equation is

$$\mathbf{P}(t) = E_\alpha(-t^\alpha), \quad t > 0.$$

Figures 9-10 show very accurate results. We also tested the algorithm with the following problem

$$\frac{d^\alpha \mathbf{P}}{dt^\alpha} + \mathbf{P} = H(t) - H(t-1), \quad P(0^+) = 0,$$

where $H(t)$ is the Heaviside step-function. The picture is identical to that in Caputo (Rend. Phys. Acc. Lincei, v. 4, pp. 89-98, 1993) where the inverse Laplace transform of $\frac{1}{s(1+s^\alpha)}$ is computed. $\alpha = 1$ corresponds to the Debye model. Presently [5], a second-order accurate code is being tested that solves the scaled 1D signaling problem, where $u(0, t) = f(t)$, $(x, t) > 0$, and $u = v = w = 0$ at $t = 0$

$$\begin{aligned} u_t + w_t &= v_x, \quad v_t = u_x \\ \frac{2 \sin \pi \alpha}{\pi} \int_0^\infty \psi(x, \xi, t) d\xi + \eta w(x, t) &= \kappa u(x, t) \\ \psi_t(x, \xi, t) + \xi^2 \psi(x, \xi, t) &= \xi^{2\alpha-1} w_t(x, t); \quad \psi(\xi, 0) = 0; \quad 0 \leq \xi < \infty, \end{aligned}$$

where $\eta = (T_p/\tau)^\alpha$ and $\kappa = \eta(\frac{\epsilon_s}{\epsilon_\infty} - 1)$.

Finally, a numerical implementation within a fourth-order accurate FD-TD scheme is given in paper [6] for the the impedance boundary condition at a planar interface, separating

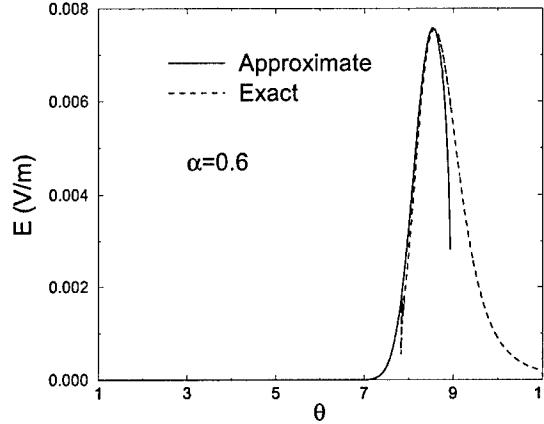


Figure 8: Same as Figure 7.

a homogeneous lossy half-space (of conductivity σ and permittivity ϵ) from free-space, which relates tangential Electric and Magnetic field components as follows:

$$E_{tan} = \left(\frac{\eta_0}{\sqrt{\epsilon_r}} \delta(t) + \zeta(t) \right) * H_{tan}, \quad (15)$$

where η_0 is the impedance of free-space, ϵ_r is the relative permittivity of the lossy half-space, $\delta(t)$ is the delta function, and $\zeta(t)$ is the time-domain impedance function. Previously [1] we have derived approximations with an error bound of $Z(t) = \left(\frac{\eta_0}{\sqrt{\epsilon_r}} \delta(t) + \zeta(t) \right)$ so the convolution can be performed accurately and quickly since the approximants are damped exponentials. Therefore, the remaining obstacle to discretizing (15) to the order of accuracy of the fourth-order scheme is to correctly extrapolate the tangential magnetic field to the boundary node which is an electric field node. For this purpose we derived a fifth-order accurate extrapolation formula which we are now testing against a fourth-order extrapolation of the form (node 0 is the boundary node where (15) is to be implemented)

$$\frac{E_0^{n+1} + E_0^n}{2} = Z^{approx} * \left(\frac{105}{48} H_{1/2} - \frac{35}{16} H_{3/2} + \frac{21}{16} H_{5/2} - \frac{15}{48} H_{7/2} \right)$$

where the r.h.s. convolution is time-centered at $t_n + \frac{\Delta t}{2}$.

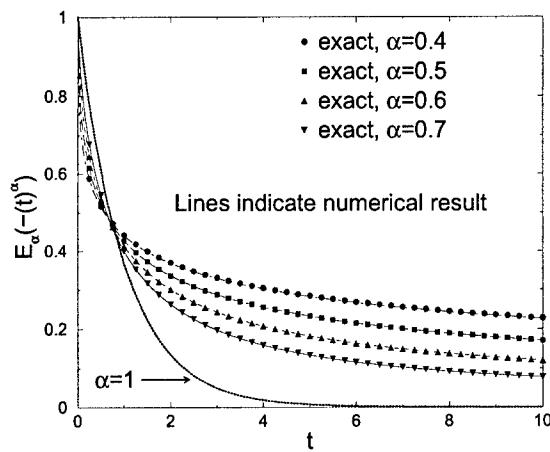


Figure 9: Numerical solution of $\frac{d^\alpha P}{dt^\alpha} + P = 0, \quad P(0^+) = 1$ for various values of α .

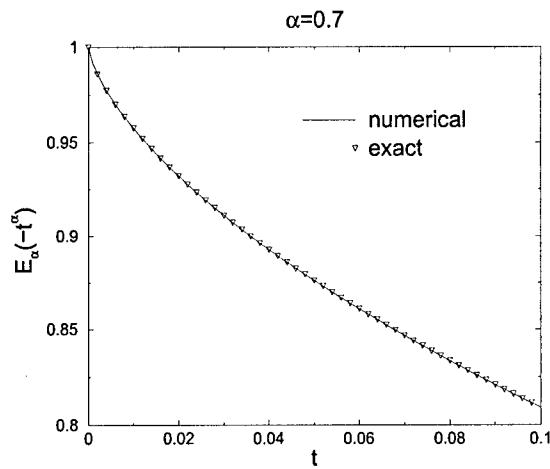


Figure 10: Detail of Figure 9 for $\alpha = 0.7$.

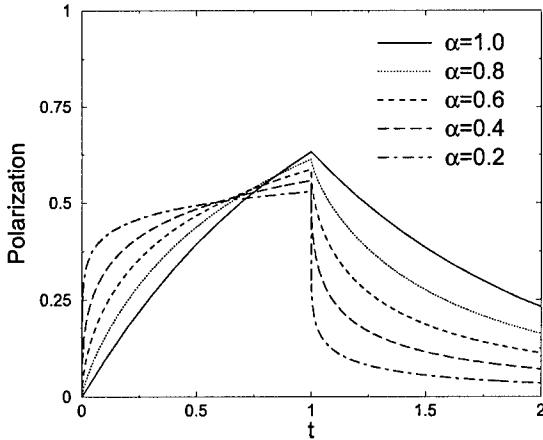


Figure 11: The polarization response of a Cole-Cole dielectric subjected to a rectangular-pulse Electric field.

2 Conference Papers and Technical Reports

The following conference paper was not reported before to AFOSR, we include it here for accuracy: “Subgridding a Fourth-Order FD-TD Scheme for Maxwell’s Equations,” (with A. Yefet), Fourth International Workshop Proceedings on Computational Electromagnetics in the Time-Domain: TLM, FDTD and Related Techniques, pp. 39-45, Nottingham, UK, 2001.

“Long-Time Behavior of the Unsplit PML, (with E. Becache and S. D. Gedney), Mathematical and Numerical Aspects of Wave Propagation WAVES 2003, pp. 120-124, Springer-Verlag (July 2003).

3 Presentations

January 2002: AFOSR Annual Electromagnetics Workshop, San Antonio, TX. Title: “A numerical and analytical study of the perfectly matched layer for Maxwell’s equations in cylindrical coordinates.”

February 2002: NJIT Applied Mathematics Colloquium, Newark, NJ. Title: Absorbing boundary conditions for the time-dependent Maxwell equations.”

June 2002: NSF-CBMS Regional Conference in the Mathematical Sciences, Numerical Methods in Forward and Inverse Electromagnetic Scattering, Golden, CO. Title: "A review of the perfectly matched layer ABC and some new results."

August 2002: NASA Langley Research Center (ICASE), Hampton, VA. Title: "On the long-time behavior of unsplit perfectly matched layers."

August 2002: Projets Estime Ondes Otto, INRIA-Rocquencourt, France. Title: "Some recent results on unsplit PML for electromagnetics."

January 2003: AFOSR Annual Electromagnetics Workshop, San Antonio, TX. Title: "On the long-time behavior of unsplit perfectly matched layers."

April 2003: "Analytical and Computational Methods in Electromagnetics," Spring Eastern Sectional Meeting of the American Mathematical Society, Courant Institute of Mathematical Sciences, NY. Title: "On the long-time behavior of unsplit perfectly matched layers."

June 2003: Department of Mathematics, University of Crete, Heraklion, Greece. Title: "Absorbing Layer Boundary Conditions for the Numerical Solution of the Time-Dependent Maxwell Equations in Open Domains."

July 2003: Sixth International Conference on Mathematical and Numerical Aspects of Wave Propagation, Jyvaskyla, Finland. Title: "Long-Time Behavior of the Unsplit PML."

July 2003: Department of Mathematical Sciences, Summer Faculty Seminar Series, New Jersey Institute of Technology, Newark, NJ. Title: "Absorbing Boundary Conditions for Numerical Solution of the Time-Dependent Maxwell Equations in Open Domains."

October 2003: Program in Applied Mathematics Colloquia Series, University of Arizona, Tucson, AZ. Title: Absorbing Boundary Conditions for Numerical Solution of the Time-Dependent Maxwell Equations in Open Domains."

January 2004: AFOSR Annual Electromagnetics Workshop, San Antonio, TX. Title: "Modeling Propagation of Time-Domain Pulses in Cole-Cole Dielectrics."

April 2004: Department of Mathematics Colloquia Series, University of New Mexico, Albuquerque, NM. Title: "Two Problems in Computational Electromagnetics."

June 2004: Department of Mathematical Sciences, Summer Faculty Seminar Series, New Jersey Institute of Technology, Newark, NJ. Title: "Modeling Propagation of Time-Domain Pulses in Cole-Cole Dielectrics."

July 2004: EUROEM 2004 International Conference, Magdeburg, Germany. Title: "Modeling Propagation of Time-Domain Pulses in Cole-Cole Dielectrics." Invited Presentation in Special Session titled "Time-Domain Techniques for the Transient Analysis of Complex Problems."

4 Consultative, Advisory Functions To Other Laboratories And Agencies, and Other Achievements

1. Throughout the period covered by this report the PI continued his interaction with Dr. T. M. Roberts of AFRL/SNHA on computational issues that arise in transient electromagnetic wave propagation. Particularly, I have been in communication with Dr. T. Roberts for the purpose of explaining/understanding a discrepancy between an asymptotic (large-depth) result, which predicts decay of peaks to be $\approx (depth)^{-1/3}$, and an observed decay of peaks ($\approx (depth)^{-2/5}$) obtained in experiments of electromagnetic pulse propagation in concrete whose dielectric properties are fitted to the Lorentz model of dispersion. For this collaboration I have been examining the wave hierarchies in Lorentz dielectrics exhibited by the partial differential equation for the electric field

$$\epsilon[\partial_t^2(E_{tt} - E_{zz}) + \alpha\partial_t(E_{tt} - E_{zz})] + (E_{tt} - \beta^2 E_{zz}) = 0$$

where $\epsilon = \frac{1}{(\omega_0^2 + \omega_p^2)T_p^2}$, $\alpha = \frac{T_p}{\tau}$ and $\beta = \frac{1}{\sqrt{\epsilon_s}}$ are non-dimensional parameters, T_p the incident pulse duration, and τ , ω_0 , ω_p , ϵ_s are the Lorentz medium parameters fitted to experimental dielectric data for concrete with $\epsilon_\infty = 1$. For the particular experimental setup, $\epsilon \approx 4.5 \times 10^{-4}$ (small), and $\alpha \approx 3.3$ (large) and $\beta \approx 0.6$. Using the method of singular perturbations I have derived the following partial differential equation (valid for $\frac{t}{\epsilon} \gg 1$ and $\alpha \approx \epsilon^{-1/3}$)

$$E_t + \beta E_z = \gamma E_{zz} - \delta E_{zzz}$$

where γ and δ depend on the parameters described above.

2. After being presented with problems of interest to Dr. R. E. Peterkin, High Power Microwave Division, Air Force Research Laboratory, Kirtland AFB I completed (November 2001) and forwarded to him a small manuscript titled "Artificial Dissipation for Damping High-Frequency Numerical Noise in FD-TD Simulations."
3. Visiting Scientist (no-cost consultant), ICASE, NASA Langley Research Center, Hampton, VA. July-August 2002.
4. Organized a mini-symposium at the May 2004 Frontiers in Applied and Computational Mathematics Conference, entitled: Computational Wave Propagation, New Jersey Institute of Technology, Newark, NJ
5. Visiting Scientist, Projet Ondes, INRIA INRIA-Rocquencourt. Visit funded by a US-France Cooperative Research Grant to Prof. Jan Hesthaven (Brown University). July 2004.

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